DEVELOPMENT OF RAINFALL-RUNOFF FORECAST MODEL

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ABSTRACT
In recent time, Artificial Neural Network (ANN) has been found useful in solving engineering problems; its accuracy in forecast of rainfall-runoff for tropical region was investigated in this work. Development of three-layered feed-forward model for rainfall-runoff forecast using gauge height, rainfall and evaporation rates was considered. Levenberg Marquadt and Bayesian Regularisation were used in training the models with data sets from two selected hydrological gauging stations of Benin-Owena River Basin Development Authority. Multiple Linear Regression model was also developed in order to compare its forecast accuracy with three-layered feed-forward model. The results obtained from the models were evaluated using coefficient of determination and root mean square error as performance statistics. From the results, the model showed higher coefficient of determination and lower root mean square error for the three-layered feed-forward networks. It was concluded that the three-layered feed-forward model improved the forecast accuracy of the runoff of Benin-Owena river basin than multiple linear regression model using the same hydrological condition.

Keywords: Feed forward network, Levenberg Marquadt, Bayesian Regularisation

INTRODUCTION
Rainfall-runoff forecast is a complex hydrological phenomenon. Various processes involved have parameters whose contributions can only be estimated, thereby affecting forecast accuracy of rainfall-runoff. But accurate forecast is necessary for hydrologists and water resources engineers to enhance their effectiveness in operational aspects of flood management etc. In an attempt to forecast rainfall-runoff, various models have been developed, as certain details of this natural phenomenon are not perfectly understood (Smith and Marshall, 2009). Different views of the physical processes of rainfall-runoff also explain the wide variation in different categories of models use for rainfall-runoff. Some of the useful approaches of modeling rainfall-runoff forecast considered the use of recorded data of rainfall-runoff in order to gain insight to the nonlinear process. This can be referred to as data driven approach (Nayak, Sudheer and...
Jain, 2007). Statistical method and artificial neural network (ANN) belong to the category of models that are data driven.

When rain begins to fall, the first drops of water are intercepted by the leaves and stems of vegetation which varies with the type and growth stage of the vegetation for natural catchments. As the rain continues, water reaching the ground surface infiltrates into the soil until it reaches a stage where the rate of rainfall (intensity) is greater than the infiltration capacity of the soil. Thereafter, surface puddles, ditches, and other depressions are filled, after which runoff is generated. The infiltration capacity of the soil is a function of soil texture, structure and the antecedent soil moisture content (Dekker and Ritsema, 2003). Runoff generation begins when rainfall intensity exceeds the actual infiltration capacity of the soil. This occurs as a result of various processes and sub-processes of the rainfall-runoff. How to develop a model that can accurately capture the exact relationship and contributions of hydrological and meteorological variables for a given catchments accurately remains a challenge.

In 1977, Nigeria was divided into eight hydrological areas and Benin-Owena River Basin Development Authority (BORDA) fell into hydrological area 6, named WESTERN LITTORAL. A number of rivers under the catchments area 6 include Siluko, Osse, Ogbese, Ethiope, Ofosu, Ossiomo, Oluwa, Owena and Oye. A total of 24 hydrological stations were established by the Benin-Owena River Basin Development Authority. The Benin-Owena River Basin Development Authority (BORDA) catchments area is shown in Figure 1. The two hydrological stations selected for the model development were Ikpoba and Okhunwan stations.

![Fig. 1: Hydrological map of Benin-Owena Catchment Area](image)

**THE RAINFALL-RUNOFF MODELS**

Conceptual models have been used for modeling of the hydrologic processes. In conceptual models, the internal descriptions of the various sub-processes are model attempting to represent, in a simplified way, the known physical processes. The input
(precipitation values) is divided into components that are routed through the sub-processes either to the watershed outlet as stream-flow or to the surface and deep storages or to the atmosphere as evapotranspiration. They attempt to provide reliable approximation of physical mechanisms, which determine the hydrologic cycle (Brath and Rosso, 1993). Among the conceptual models used for rainfall-runoff are Watbal model (Markus and Baker, 1994); Sacramento Soil Moisture Accounting (SAC-SMA) model and SCRR model (McCuen and Snyder, 1986). Generally, the use of conceptual model requires a great amount of information regarding the physical properties of the catchment’s area which unfortunately can be difficult to obtain.

**Statistical model:** Statistical model is also used for rainfall-runoff modeling. Statistical models generally require a data set of past observations sufficiently large to allow the system to be adequately parameterized (Morales, Ibrahim, Chen and Ryan, 2006). Such statistical models include autoregressive linear model, multiple linear regression model and moving average method among others. Multiple linear regressions establish quantitative relationship between group of predictor variables and observed response. Consider the linear regression model with single independent variable in equation 1. The linear model has the form

\[ y = X\alpha + \epsilon \]  

where \( X \) refers to regressor variable  
\( \alpha \) refers to the vector of parameter or coefficient  
\( \epsilon \) is the random disturbances.  
\( y \) is the dependent observation.

To resolve \( x \) using least square estimate, equation (2) can be used

\[ \alpha = (X^T X)^{-1} X^T y \]  

If \( y \) is a function of more than one independent variable, the matrix equations that express the relationships among the variables can be expanded to accommodate the additional variables and equation (1) becomes

\[ y = \alpha_0 + \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 \]  

\( \alpha_0, \alpha_1, \alpha_2, \alpha_3 \) are the intercept and coefficients respectively for the regressor variables: Gaugeheight, Rainfall and Evaporation rates.

**Artificial Neural Network (ANN):** Artificial Neural Network (ANN) is among the data driven approach that can be used to capture the non-linear process involve in rainfall-runoff. ANN does not require the complex nature of the underlying process under consideration to be explicitly described in any form (ASCE, 2005). ANN
approach has been used to solve problems in control, function approximation and pattern classification. The predictive potentiality of (ANN) is widely acknowledged in its applications to hydrological problems (Atiya et al., 1999). ANN emulates the biological nervous systems by distributing computations to processing units called neurons, which are densely interconnected. In Figure 2.0 various inputs to the network are represented by the symbol $X_0, X_1, X_2, \ldots, X_n$ which combine with connection weights, $W_0, W_1, W_2, \ldots, W_n$ and the bias is represented as $b_k$.

![Figure 2: Structure of a Neuron](image)

A neuron usually receives many simultaneous inputs. Weights are adaptive coefficients within the network that determine the intensity of the input's connection strength. These strengths can be modified in response to various training sets and according to a network's specific topology or through its learning rules. The summation function is found by multiplying each component of the input vector by the corresponding component of the weight vector and then adding up all the products that is,

$$u_i = \sum_j w_{ij}x_j + b_k$$

$u_i$ represents the total input to hidden units $j$ as a linear function of outputs of $x_i$ and $w_{ij}$ is the weight from node $i$ connecting to node $j$ with bias $b_k$. 

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Three-Layered Feedforward Model: Among various architectures of Artificial Neural Network, the feed-forward model is the mostly used network architecture for handling the dynamics of rainfall-runoff, (Hsu, Gupta and Sorooshian, 1995). Multiple layers of neurons with non-linear transfer functions allow the network to learn nonlinear and linear relationships between input and output vectors (Jang, Sun and Mizutani, 1997). In this study, three-layered feed-forward model was used to forecast rainfall-runoff for tropical region using Benin-Owena river basin as the case study. In Ikpoba and Okhunwan hydrological stations selected from Benin-Owena river basin, the same hydrological variables namely: gauge height, rainfall and evaporation rates were used as input variable in developing the model. The model architecture is shown in Figure 3. Amidst the training functions used with feed-forward network, Levenberg Marquadt and Bayesian Regularisation give optimum performance (Aqil, Kita, Yano and Nishiyama, 2007).

\[ X_n = (GH_n, WD_n, RF_n, ER_n) \]  \hspace{1cm} (5)

The above represents the input variables into the network. The transfer function sigmoid was then applied to the inputs using equation (6).

\[ f(x_i) = \frac{1}{1 + e^{-x_i}} \]  \hspace{1cm} (6)

Where \( i = 1, 2...n \)

The transformed target outputs obtained from the network are:

\[ T_n = (t_1, t_2, ..., t_n) \]  \hspace{1cm} (7)

While the actual outputs from the observed is represented in

\[ Y_n = (y_1, y_2, ..., y_n) \]  \hspace{1cm} (8)

The network seeks to minimise the difference between the actual outputs and the network outputs using the mean square error in equation (9).

\[ MSE = \frac{1}{n} \sum_{i=1}^{n} (y_i - T_i)^2 \]  \hspace{1cm} (9)

The data sets were divided into training sets, validation sets and testing sets. Two types of three-layered feed-forward network model were used. Each model has one input layer, one hidden layer and one output layer. The number of neurons in each layer however differs. Three neurons were used in the input layer; the number of neurons in the hidden layer was determined by varying the number of neurons and then checking the performance of the network through root mean square error. The number of neuron in the output layer was one since only one output was required. Non-linear sigmoid function, tansig was used at the hidden layer while the output used linear function, purelin to generate a single output at the output layer. The training was done using Okhunwan station data set while Ikpoba station data set was used for testing the model.
The model architecture adopted after varying different number of neurons at the hidden layer was 3:5:1. The graph for determination of neurons in the hidden layer against root mean square error, RMSE is shown in Figure 4. Levenberg Marquadt training function, trainlm and Bayesian Regularisation, training algorithm were used with the model architecture. The three-layered feed-forward model that used Levenberg Marquadt as the training function was the ANN1 while the model that used Bayesian regularization was ANN2. During the back propagation training, the model output was compared to the observed real values and the differences as error was redistributed back into the network for adjustment that took place through the weights being adjusted repeatedly until the errors for the data sets were sufficiently minimized. Weights in the hidden layer were adjusted using equation 10

\[ w_{ji}(n+1) = w_{ji}(n) + \eta \delta_{pj} x_{pi} \]

And weights in the output layers were adjusted using equation (11):

\[ w_{kj}(n+1) = w_{kj}(n) + \eta \delta_{pk} \]

where:

- \( i = \) unit node of the input layer
- \( j = \) unit node of the hidden layer
- \( p = \) pattern in the dataset and \( k \) is related to the output layer
- \( \eta = \) learning rate
- \( \delta_{pk} \) and \( \delta_{pj} \) are error part including derivative part for output and hidden units respectively
- \( n = n^{th} \) iteration
- \( x_{pi} = (x_{p1}, x_{p2}, x_{p3} ... x_{pn}) \)

![Figure 3: Architecture of three-layered feed-forward network](image-url)
Table 1: Dataset Parameters for Training

<table>
<thead>
<tr>
<th>Statistical parameter</th>
<th>Gauge Height (cm)</th>
<th>Rainfall (mm)</th>
<th>Evaporation Rates (mm)</th>
<th>Runoff (m³/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>1.680</td>
<td>0.000</td>
<td>1.200</td>
<td>710000000</td>
</tr>
<tr>
<td>Max</td>
<td>46.000</td>
<td>10.670</td>
<td>11</td>
<td>168000000</td>
</tr>
<tr>
<td>Mean</td>
<td>17.260</td>
<td>0.762</td>
<td>2.645</td>
<td>109651376</td>
</tr>
<tr>
<td>Standard Dev.</td>
<td>16.931</td>
<td>1.601</td>
<td>1.679</td>
<td>29522803</td>
</tr>
</tbody>
</table>

Table 2: Data Set Parameters for Testing

<table>
<thead>
<tr>
<th>Statistical parameter</th>
<th>Gauge Height (cm)</th>
<th>Rainfall (mm)</th>
<th>Evaporation Rates (mm)</th>
<th>Runoff (m³/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>1.110</td>
<td>0.000</td>
<td>1.000</td>
<td>180000000</td>
</tr>
<tr>
<td>Max</td>
<td>4.150</td>
<td>1.561</td>
<td>6.210</td>
<td>626000000</td>
</tr>
<tr>
<td>Mean</td>
<td>2.402</td>
<td>0.055</td>
<td>2.585</td>
<td>86575653</td>
</tr>
<tr>
<td>Standard Dev.</td>
<td>0.724</td>
<td>0.138</td>
<td>1.147</td>
<td>86227309</td>
</tr>
</tbody>
</table>

Fig: 4: Determination of the number of hidden neurons

**PERFORMANCE EVALUATION OF THE MODEL**

For the purpose of evaluating the model, coefficient of determination and root mean square errors were chosen as performance statistics with which the model can be assessed in terms of accuracy. When the two models were subjected to the same conditions, the performance evaluation statistics were used to determine the required accuracy as stated below.
Coefficient of determination, $R^2$:
\[
R^2 = \frac{\sum_{i=1}^{n} (Q_{\text{obs}} - Q_{av})(Q_{cal} - Q_{cal-av})^2}{\sum_{i=1}^{n} (Q_{obs} - Q_{av})^2 \sum_{i=1}^{n} (Q_{cal} - Q_{cal-av})^2}
\] (12)

Root Mean Square Error:
\[
\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n} (Q_{\text{obs}} - Q_{cal})^2}{n}}
\] (13)

Where $n$ is the number of observations, $Q_{\text{obs}}$ and $Q_{cal}$ are the observed and calculated values, and $Q_{av}$, $Q_{cal-av}$ are the mean of the observed and calculated values respectively.

The model forecast results showed closeness between the model predictions for the three-layered feed-forward network models, ANN1 and ANN2. The statistical performance of the models for training is shown on table 3. For the training set, ANN1 produced the highest coefficient of determination, $R^2 = 0.9924$ and least root mean square error, RMSE = 3.318 among the models. The multiple linear regression (MLR) model gave the highest root mean square error, RMSE = 9.451. The graphs of the three-layered feed-forward network model for ANN1 and ANN2 are shown in figures 5 and 6 respectively. The graph of the MLR model is shown in figure 7.

The statistical performance of the models for testing set is shown on table 4. The three-layered feed-forward network model that used Levenberg Marquadt, trainlm as training function, ANN1 performed better in terms of accuracy above the feed-forward network model that used Bayesian Regularisation, trainbr as training function, ANN2. The ANN1 model gave the coefficient of determination of 0.9851 which is the highest for the testing set and minimum root mean square of 3.534 which is the least for the testing set. The scatter plots of the three-layered feed-forward models for ANN1 and ANN2 are shown in figures 8 and 9 respectively. The scatter diagram of the multiple linear regression models is figure 10. MLR did not demonstrate good accuracy as most of the predictions fail to fall on the regression line. Using the two statistical indices, it can be concluded that the artificial neural network has improved accuracy over multiple linear regression model under the same hydrological condition. Based on its performance, three-layered-feed-forward model can be adopted for operational purposes.

**Table 3: Performance evaluation for testing data set**

<table>
<thead>
<tr>
<th>ANN Model</th>
<th>Coefficient of Determination, $R^2$</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANN1</td>
<td>0.9924</td>
<td>3.318</td>
</tr>
<tr>
<td>ANN2</td>
<td>0.9844</td>
<td>3.370</td>
</tr>
<tr>
<td>MLR</td>
<td>0.7564 9.451</td>
<td></td>
</tr>
</tbody>
</table>
Table 4: Performance statistical table for testing set

<table>
<thead>
<tr>
<th>ANN Model</th>
<th>$R^2$</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANN1</td>
<td>0.9851</td>
<td>3.534</td>
</tr>
<tr>
<td>ANN2</td>
<td>0.9428</td>
<td>6.923</td>
</tr>
<tr>
<td>MLR</td>
<td>0.7323</td>
<td>10.18</td>
</tr>
</tbody>
</table>

Figure 5: ANN1 graph using Ikpoba dataset

Figure 6: ANN2 graph using Ikpoba dataset
Figure 7: MLR graph using Ikpoba dataset

Figure 8: ANN1 graph using Okhunwan station

Figure 9: ANN2 graph using Okhunwan station
REFERENCES


Figure 10: MLR graph using Okhunwan station